Beal City High School
Precalculus Curriculum and Alignment

UNIT 1 Linear and Quadratic Functions (Chapter 1)

1. Points and lines (1-1, 2, 3)
2. Linear functions and models (1-4)
3. The complex numbers (1-5)
4. Solving quadratic equations (1-6)
5. Quadratic functions and their graphs (1-7)
6. Quadratic models (1-8)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

N-CN.1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.

- Know that every number is a complex number of the form a + bi, where a and b are real numbers.
- Know that the complex number $i^2 = -1$.

N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

- Apply the fact that the complex number $i^2 = -1$.
- Use the associative, commutative, and distributive properties, to add, subtract, and multiply complex numbers.

N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

- Given a complex number, find its conjugate and use it to find quotients of complex numbers.
- Find the magnitude (length), modulus (length) or absolute value (length), of the vector representation of a complex number.

N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.

- Solve quadratic equations with real coefficients that have solutions of the form a + bi and a – bi.

N-CN.8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.

- Use polynomial identities to write equivalent expressions in the form of complex numbers

N-CN.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

- Understand The Fundamental Theorem of Algebra, which says that the number of complex solutions to a polynomial equation is the same as the degree of the polynomial. Show that this is true for a quadratic polynomial.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines. (Write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros. Given a quadratic function explain the meaning of the zeros of the function. That is if f(x) = (x – c) (x – a) then f(a) = 0 and f(c) = 0. Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression (x – a) (x – c), a and c correspond to the x-intercepts (if a and c are real).)
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (Write expressions in equivalent forms by completing the square to convey the vertex form, to find the maximum or minimum value of a quadratic function, and to explain the meaning of the vertex.)

A-APR.4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

- Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc .
- Prove polynomial identities by showing steps and providing reasons.
- Illustrate how polynomial identities are used to determine numerical relationships.

A-CED.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

- Create linear, quadratic, rational and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.

A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

A-REI.4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. (Transform a quadratic equation written in standard form to an equation in vertex form (x - p) = q 2 by completing the square. Derive the quadratic formula by completing the square on the standard form of a quadratic equation.)

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. (Solve quadratic equations in one variable by simple inspection, taking the square root, factoring, and completing the square. Understand why taking the square root of both sides of an equation yields two solutions. Use the quadratic formula to solve any quadratic equation, recognizing the formula produces all complex solutions. Write the solutions in the form a ± bi , where a and b are real numbers. Explain how complex solutions affect the graph of a quadratic equation.)

A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

- Solve systems of equations using the elimination method (sometimes called linear combinations).
- Solve a system of equations by substitution (solving for one variable in the first equation and substitution it into the second equation).

A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

- Solve systems of equations using graphs.

A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.

- Solve a system containing a linear equation and a quadratic equation in two variables (conic sections possible) graphically and symbolically.

A-REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

- Write a system of linear equations as a single matrix equation.

F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

- Given a function, identify key features in graphs and tables including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Given the key features of a function, sketch the graph.

F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

- Given the graph of a function, determine the practical domain of the function as it relates to the numerical relationship it describes.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

G-GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

- Using slope, prove lines are parallel or perpendicular
- Find equations of lines based on certain slope criteria such as; finding the equation of a line parallel or perpendicular to a given line that passes through a given point.

UNIT 2 Polynomial Functions and Inequalities (Chapters 2 and 3)

1. Polynomials (2-1)
2. Division (2-2)
3. Graphing polynomial functions (2-3)
4. Finding maximums and minimums (2-4)
5. Linear Inequalities; absolute value (3-1)
6. Polynomial inequalities (3-2, 3)
7. Linear programming (3-4)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

N-CN.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

- Understand The Fundamental Theorem of Algebra, which says that the number of complex solutions to a polynomial equation is the same as the degree of the polynomial. Show that this is true for a quadratic polynomial.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines. (Write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros. Given a quadratic function explain the meaning of the zeros of the function. That is if f(x) = (x – c) (x – a) then f(a) = 0 and f(c) = 0. Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression (x – a) (x – c), a and c correspond to the x-intercepts (if a and c are real).)

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (Write expressions in equivalent forms by completing the square to convey the vertex form, to find the maximum or minimum value of a quadratic function, and to explain the meaning of the vertex.)

A-APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- Understand the definition of a polynomial.
- Understand the concepts of combining like terms and closure.
- Add, subtract, and multiply polynomials and understand how closure applies under these operations.

A-APR.2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

- Understand and apply the Remainder Theorem.
- Understand how this standard relates to A.SSE.3a.
- Understand that a is a root of a polynomial function if and only if x-a is a factor of the function.

A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- Find the zeros of a polynomial when the polynomial is factored.
- Use the zeros of a function to sketch a graph of the function.

A-APR.6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

- Rewrite rational expressions by using factoring, long division, or synthetic division. Use a computer algebra system for complicated examples to assist with building a broader conceptual understanding.

A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.

A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

- Write and use a system of equations and/or inequalities to solve a real world problem. Recognize that the equations and inequalities represent the constraints of the problem. Use the Objective Equation and the Corner Principle to determine the solution to the problem.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

- Given a function, identify key features in graphs and tables including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Given the key features of a function, sketch the graph.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

UNIT 3 Functions (Chapter 4)

1. Functions (4-1)
2. Operations on Functions (4-2)
3. Graph Transformations (4-3, 4)
4. Inverse Functions (4-5)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

- Use the definition of a function to determine whether a relationship is a function given a table, graph or words.
- Given the function f(x), identify x as an element of the domain, the input, and f(x) is an element in the range, the output.
- Know that the graph of the function, f, is the graph of the equation y=f(x).

F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- When a relation is determined to be a function, use f(x) notation.

- Evaluate functions for inputs in their domain.
- Interpret statements that use function notation in terms of the context in which they are used.

F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

- Given a function, identify key features in graphs and tables including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Given the key features of a function, sketch the graph.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-BF.1. Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context. (From context, write an explicit expression, define a recursive process, or describe the calculations needed to model a function between two quantities.)
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (Combine standard function types, such as linear and exponential, using arithmetic operations.)
c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- Identify, through experimenting with technology, the effect on the graph of a function by replacing f(x) with f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative).
- Given the graphs of the original function and a transformation, determine the value of (k).
- Recognize even and odd functions from their graphs and equations.

F-BF.4. Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$.* (Solve a function for the dependent variable and write the inverse of a function by interchanging the values of the dependent and independent variables.)
b. (+) Verify by composition that one function is the inverse of another. (Verify that one function is the inverse of another by illustrating that f-1(f(x)) = f(f-1(x)) = x.)
c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
d. (+) Produce an invertible function from a non-invertible function by restricting the domain. (Find the inverse of a function that is not one-to-one by restricting the domain.)

UNIT 4 Conic Sections (Chapter 6)

1. Circles (6-2)
2. Ellipses (6-3)
3. Hyperbolas (6-4)
4. Parabolas (6-5)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

- Solve a system containing a linear equation and a quadratic equation in two variables (conic sections possible) graphically and symbolically.

F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

- Given a function, identify key features in graphs and tables including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Given the key features of a function, sketch the graph.

G-C.4. (+) Construct a tangent line from a point outside a given circle to the circle.

- Construct a tangent line from a point outside a given circle to the circle.

G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- Use the Pythagorean Theorem to derive the equation of a circle, given the center and the radius.
- Given an equation of a circle, complete the square to find the center and radius of a circle.

G-GPE.2. Derive the equation of a parabola given a focus and directrix.

- Given a focus and directrix, derive the equation of a parabola.
- Given a parabola, identify the vertex, focus, directrix, and axis of symmetry, noting that every point on the parabola is the same distance from the focus and the directrix.

G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

- Given the foci, derive the equation of an ellipse, noting that the sum of the distances from the foci to any fixed point on the ellipse is constant, identifying the major and minor axis.
- Given the foci, derive the equation of a hyperbola, noting that the absolute value of the differences of the distances form the foci to a point on the hyperbola is constant, and identifying the vertices, center, transverse axis, conjugate axis, and asymptotes.

UNIT 5 Trigonometric Functions (Chapter 7)

1. Measurements of angles (7-1)
2. Sectors of circles (7-2)
3. Sine and cosine functions (7-3)
4. The other trigonometric functions (7-5)
5. Graphs of the trigonometric functions (7-4)
6. Inverse trigonometric functions (7-6)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

- Given a function, identify key features in graphs and tables including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Given the key features of a function, sketch the graph.

F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

- Know that if the length of an arc subtended by an angle is the same length as the radius of the circle, then the measure of the angle is 1 radian.
- Know that the graph of the function, f, is the graph of the equation y=f(x).

F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

- Explain how radian measures of angles rotated counterclockwise in a unit circle are in a one-to-one correspondence with the nonnegative real numbers, and that angles rotated clockwise in a unit circle are in a one-to-one correspondence with the non-positive real numbers.

F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

- Use 30º-60º-90º and 45º-45º-90º triangles to determine the values of sine, cosine, and tangent.

F-TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

- Use the unit circle and periodicity to find values of sine, cosine, and tangent for any value of θ, such as $\pi + \theta$, $2\pi + \theta$, where θ is a real number.

- Use the values of the trigonometric functions derived from the unit circle to explain how trigonometric functions repeat themselves.
- Use the unit circle to explain that f(x) is an even function if f(-x) = f(x), for all x, and an odd function if f(-x) = -f(x). Also know that an even function is symmetric about the y-axis.

F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

- Use sine and cosine to model periodic phenomena such as the ocean's tide or the rotation of a Ferris wheel.
- Given the amplitude; frequency; and midline in situations or graphs, determine a trigonometric function used to model the situation.

F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

- Know that the inverse for a trigonometric function can be found by restricting the domain of the function so it is always increasing or decreasing.

F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

- Use the unit circle to prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.
- Given the value of the $\sin(\theta)$ or $\cos(\theta)$, use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to calculate other trigonometric ratios.

G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

- Use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius, identifying the constant of proportionality as the radian measure of the angle.
- Find the arc length of a circle.
- Using similarity, derive the formula for the area of a sector.
- Find the area of a sector in a circle.

UNIT 6 Trigonometric Equations and Triangle Trigonometry (Chapter 9)

1. Solving trigonometric equations (8-1)
2. Solving right triangles (9-1)
3. Area of a triangle (9-2)
4. The Law of Sines (9-3)
5. The Law of Cosines (9-4)
6. Navigation and Surveying Applications of Trigonometry (9-5)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.

- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

- Use the inverse of trigonometric functions to solve equations that arise in real-world contexts.
- Use technology to evaluate the solutions to the inverse trigonometric functions, and interpret their meaning in terms of the context.

G-SRT.9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

- For a triangle that is not a right triangle, draw an auxiliary line from a vertex, perpendicular to the opposite side and derive the formula, A=½ ab sin (C), for the area of a triangle, using the fact that the height of the triangle is, h=a sin(C).

G-SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

- Using trigonometry and the relationship among sides and angles of any triangle, such as sin(C)=(h/a) , prove the Law of Sines.
- Using trigonometry and the relationship among sides and angles of any triangle and the Pythagorean Theorem to prove the Law of Cosines.
- Use the Laws of Sines to solve problems.
- Use the Laws of Cosines to solve problems.

G-SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

- Understand and apply the Law of Sines and the Law of Cosines to find unknown measures in right triangles.
- Understand and apply the Law of Sines and the Law of Cosines to find unknown measures in non-right triangles.

UNIT 7 Trigonometric Identities and Formulas (Chapters 8 and 10)

1. Trigonometric identities (8-4)
2. Sine and cosine addition and subtraction formulas (10-1)
3. Tangent addition and subtraction and difference formulas (10-2)
4. Double-angle formulas (10-3)
5. Half-angle formulas (10-4)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

F-TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

- Prove the addition and subtraction formulas $\sin(\acute{\alpha}\pm\beta)$, $\cos(\acute{\alpha}\pm\beta)$, and $\tan(\acute{\alpha}\pm\beta)$.
- Use the addition and subtraction formulas to determine exact trigonometric values such as $\sin(75^o)$ or $\cos(12)$.

UNIT 8 Polar Coordinates, Vectors and Matrices (Chapters 11, 12 and 14)

1. Polar coordinates and graphs (11-1)
2. Geometric representation of complex numbers (11-2)
3. Powers of complex numbers (11-3)
4. Geometric representation of vectors (12-1)
5. Algebraic representation of vectors (12-2)
6. Motion applications of vectors (12-3)
7. Dot product (12-4)
8. Determinants (12-7, 8)
9. Matrix addition and scalar multiplication (14-1)
10. Matrix multiplication (14-2)
11. Matrix properties and inverse (14-3)
12. Matrix applications (14-4, 5)
13. Transformation matrices (14-6)

CCSS:

N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- Interpret units in the context of the problem
- When solving a multi-step problem, use units to evaluate the appropriateness of the solution.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays

N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.

- Determine and interpret appropriate quantities when using descriptive modeling.

N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

- Determine the accuracy of values based on their limitations in the context of the situation.

N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

- Find the magnitude (length), modulus (length) or absolute value (length), of the vector representation of a complex number.

N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

- Transform complex numbers in a complex plane from rectangular to polar form and vice versa,
- Know and explain why both forms, rectangular and polar, represent the same number.

N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°.

- Geometrically show addition, subtraction, and multiplication of complex numbers on the complex coordinate plane.
- Geometrically show that the conjugate of complex numbers in a complex plane is the reflection across the x-axis.
- Evaluate the power of a complex number, in rectangular form, using the polar form of that complex number.

N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

- Calculate the distance between values in the complex plane as the magnitude, modulus, of the difference, and the midpoint of a segment as the average of the coordinates of its endpoints.

N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $\|\boldsymbol{v}\|$, v).

- Know that a vector is a directed line segment representing magnitude and direction.
- Use the appropriate symbol representation for vectors and their magnitude.

N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

- Find the component form of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point, therefore placing the initial point of the vector at the origin.

N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

- Solve problems such as velocity and other quantities that can be represented using vectors.

N-VM.4. (+) Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. (Know how to add vectors head to tail, using the horizontal and vertical components, and by finding the diagonal formed by the parallelogram.)

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. (Understand that the magnitude of a sum of two vectors is not the sum of the magnitudes unless the vectors have the same heading or direction.)

c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. (Know how to subtract vectors and that vector subtraction is defined much like subtraction of real numbers, in that v – w is the same as v + (–w), where –w is the additive inverse of w. The opposite of w, -w, has the same magnitude, but the direction of the angle differs by 180. Represent vector subtraction on a graph by connecting the vectors head to tail in the correct order and using the components of those vectors to find the difference.)

N-VM.5. (+) Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. (Represent scalar multiplication of vectors on a graph by increasing or decreasing the magnitude of the vector by the factor of the given scalar. If the scalar is less than zero, the new vector's direction is opposite the original vector's direction. Represent scalar multiplication of vectors using the component form, such as c(vx, vy) = (cvx, cvy).)

b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|c\boldsymbol{v}\| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$). (Find the magnitude of a scalar multiple, cv, is the magnitude of v multiplied by the factor of the |c|. Know when c > 0, the direction is the same, and when c < 0, then the direction of the vector is opposite the direction of the original vector.)

N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

- Represent and manipulate data using matrices, e.g., to organize merchandise, keep total sales, costs, and using graph theory and adjacency matrices to make predictions.

N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

- Multiply matrices by a scalar, e.g., when the inventory of jeans for July is twice that for January.

N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

- Know that the dimensions of a matrix are based on the number of rows and columns.
- Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

- Understand that matrix multiplication is not commutative, AB BA, however it is associative and satisfies the distributive properties.

N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

- Identify a zero matrix and understand that it behaves in matrix addition, subtraction, and multiplication, much like 0 in the real numbers system.
- Identify an identity matrix for a square matrix and understand that it behaves in matrix multiplication much like the number 1 in the real number system.
- Find the determinant of a square matrix, and know that it is a nonzero value if the matrix has an inverse.
- Know that if a matrix has an inverse, then the determinant of a square matrix is a nonzero value.

N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

- To translate the vector AB, where A(1,3) and B(4,9), 2 units to the right and 5 units up, perform the following matrix multiplication.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 9 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 8 & 14 \\ 1 & 1 \end{bmatrix}$$

N-VM.12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

- Given the coordinates of the vertices of a parallelogram in the coordinate plane, find the vector representation for two adjacent sides with the same initial point. Write the components of the vectors in a 2x2 matrix and find the determinant of the 2x2 matrix. The absolute value of the determinant is the area of the parallelogram. (This is called the dot product of the two vectors.)

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients. (Identify the different parts of the expression and explain their meaning within the context of a problem.)

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* (Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.)

A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

- Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.
- Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely.
- Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- Solve linear equations in one variable, including coefficients represented by letters.
- Solve linear inequalities in one variable, including coefficients represented by letters.

A-REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

- Write a system of linear equations as a single matrix equation.

A-REI.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

- Find the inverse of the coefficient matrix in the equation, if it exits. Use the inverse of the coefficient matrix to solve the system. Use technology for matrices with dimensions 3 by 3 or greater.
- Find the dimension of matrices.
- Understand when matrices can be multiplied.
- Understand that matrix multiplication is not commutative.
- Understand the concept of an identity matrix.
- Understand why multiplication by the inverse of the coefficient matrix yields a solution to the system (if it exists).

Precalculus—12th grade

Units		Common Core Standards	Vocabulary	Pacing
Unit 1: Linear and Quadratic Functions	Chapter 1, Section 1 Chapter 1, Section 2 Chapter 1, Section 3 Chapter 1, Section 4 Chapter 1, Section 5 Chapter 1, Section 6 Chapter 1, Section 7 Chapter 1, Section 8	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **N-CN.1.** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and*b* real. **N-CN.3.** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **N-CN.7.** Solve quadratic equations with real coefficients that have complex solutions. **N-CN.8.** (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.* **N-CN.9.** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.* **A-SSE.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A-APR.4.** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.* **A-CED.1.** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **A-CED.2.** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Assessments:** Multiple quizzes Final test	Point Line Distance Midpoint Slope System of linear equations Parallel and perpendicular lines Perpendicular bisector Median Altitude Linear function Complex numbers Conjugate Factoring Quadratic formula Complete the square Parabola Discriminant Line of symmetry Vertex Quadratic regression	11 days

Precalculus—12th grade

Units		Common Core Standards (cont.)	Vocabulary	Pacing
Unit 1: Linear and Quadratic Functions (cont.)		**A-REI.4.** Solve quadratic equations in one variable. **A-REI.5.** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A-REI.6.** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **A-REI.7.** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. **A-REI.8. (+)** Represent a system of linear equations as a single matrix equation in a vector variable. **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **F-IF.5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **F-IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **G-GPE.5.** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).		

Precalculus—12th grade

Units		Common Core Standards	Vocabulary	Pacing
Unit 2: Polynomial Functions and Inequalities	Chapter 2, Section 1 Chapter 2, Section 2 Chapter 2, Section 3 Chapter 2, Section 4 Chapter 3, Section 1 Chapter 3, Section 2 Chapter 3, Section 3 Chapter 3, Section 4	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **N-CN.9.** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **A-SSE.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A-APR.1.** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A-APR.2.** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **A-APR.3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **A-APR.6.** Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. **A-CED.2.** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **Assessments:** Multiple quizzes Final test	Polynomial Zeros Synthetic substitution The Remainder Theorem Long division Synthetic division Sign analysis Maximum/minimum Inequality Absolute value Polynomial inequalities Linear programming	17 days

Precalculus—12th grade

Units		Common Core Standards (cont.)	Vocabulary	Pacing
Unit 2: Polynomial Functions and Inequalities (cont.)		**A-CED.3.** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **F-IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.		

Precalculus—12th grade

Units		Common Core Standards	Vocabulary	Pacing
Unit 3: Functions	Chapter 4, Section 1 Chapter 4, Section 2 Chapter 4, Section 3 Chapter 4, Section 4 Chapter 4, Section 5	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **F-IF.1.** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). **F-IF.2**. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **F-IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-BF.1.** Write a function that describes a relationship between two quantities.	Function Vertical-line test Domain/range Zeros Operations on functions Composition of functions Parent functions Reflecting graphs Symmetry Odd/even functions Stretching graphs Translating graphs Inverse functions	15 days
		Assessments: Multiple quizzes Final test		

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Units		Common Core Standards (cont.)	Vocabulary	Pacing
Unit 3: Functions (cont.)		**F-BF.3.** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. **F-BF.4.** Find inverse functions.		

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Units		Common Core Standards	Vocabulary	Pacing
Unit 4: Conic Sections	Chapter 6, Section 2 Chapter 6, Section 3 Chapter 6, Section 4 Chapter 6, Section 5	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **A-CED.2.** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **A-REI.7.** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **G-C.4.** (+) Construct a tangent line from a point outside a given circle to the circle. **G-GPE.1.** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G-GPE.2.** Derive the equation of a parabola given a focus and directrix. **G-GPE.3. (+)** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	Conic sections Cones Circles Center/radius Ellipses Foci Vertices/co-vertices Major/minor axis Hyperbolas Asymptotes Parabolas Directrix	13 days
		Assessments: Multiple quizzes Final test		

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Units		Common Core Standards	Vocabulary	Pacing
Unit 5: Trigonometric Functions	Chapter 7, Section 1 Chapter 7, Section 2 Chapter 7, Section 3 Chapter 7, Section 5 Chapter 7, Section 4 Chapter 7, Section 6	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **A-CED.2.** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. **F-IF.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-TF.1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.2.** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF.3.** (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. **Assessments:** Multiple quizzes Final test	Degrees/minutes/seconds Radians Coterminal Sector Arc length Sine Cosine Unit circle Reference angle Tangent Secant Cotangent Secant Periodic Cycle Period Amplitude Frequency Inverse trig function Domain restriction One-to-one function Many-to-one function	24 days

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Units		Common Core Standards (cont.)	Vocabulary	Pacing
Unit 5: Trigonometric Functions (cont.)		**F-TF.4.** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **F-TF.5.** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **F-TF.6.** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **F-TF.8.** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. **G-C.5.** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.		

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Units		Common Core Standards	Vocabulary	Pacing
Unit 6: Trigonometric Equations and Triangle Trigonometry	Chapter 8, Section 1 Chapter 9, Section 1 Chapter 9, Section 2 Chapter 9, Section 3 Chapter 9, Section 4 Chapter 9, Section 5	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **F-TF.7.** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **G-SRT.9.** (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G-SRT.10.** (+) Prove the Laws of Sines and Cosines and use them to solve problems. **G-SRT.11.** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **Assessments:** Multiple quizzes Trig Star competition Final test	Solving trig equations Inclination Slope Trig ratios Pythagorean Theorem Area of a triangle Law of Sines Ambiguous case Law of Cosines Bearing Navigating Surveying	20 days

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Units		Common Core Standards	Vocabulary	Pacing
Unit 7: Trigonometric Identities and Formulas	Chapter 8, Section 4 Chapter 10, Section 1 Chapter 10, Section 2 Chapter 10, Section 3 Chapter 10, Section 4	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **F-TF.9.** (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	Reciprocal identities Ratio identities Pythagorean identities Sine and cosine addition and subtraction formulas Tangent addition and subtraction formulas Double-angle formulas Half-angle formulas	14 days
		Assessments: Multiple quizzes Final test		

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Units		Common Core Standards	Vocabulary	Pacing
Unit 8: Polar Coordinates, Vectors, and Matrices	Chapter 11, Section 1 Chapter 11, Section 2 Chapter 11, Section 3 Chapter 12, Section 1 Chapter 12, Section 2 Chapter 12, Section 3 Chapter 12, Section 4 Chapter 12, Section 7 Chapter 12, Section 8 Chapter 14, Section 1 Chapter 14, Section 2 Chapter 14, Section 3 Chapter 14, Section 4 Chapter 14, Section 5 Chapter 14, Section 6	**N-Q.1.** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3.** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **N-CN.3.** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **N-CN.4.** (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN.5.** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°. **N-CN.6.** (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **N-VM.1.** (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, $\|\boldsymbol{v}\|$, $\|\boldsymbol{v}\|$, *v*). **N-VM.2.** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **N-VM.3.** (+) Solve problems involving velocity and other quantities that can be represented by vectors. **N-VM.4.** (+) Add and subtract vectors. **N-VM.5.** (+) Multiply a vector by a scalar. **N-VM.6.** (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **N-VM.7.** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **Assessments:** Multiple quizzes Final test	Polar coordinates Rectangular coordinates Complex number plane Absolute value cis De Moivre's Theorem Vector Magnitude Direction Vector sum Negative of a vector Vector subtraction Multiples of a vector Scalar Scalar multiplication Component form Vector equation Constant velocity Dot product Determinant Cramer's rule Matrix Matrix operations Dimensions Commutative Associative Distributive	30 days

Precalculus—12th grade

Units		Common Core Standards (cont.)	Vocabulary(cont.)	Pacing
Unit 8: Polar Coordinates, Vectors, and Matrices (cont.)		**N-VM.8.** (+) Add, subtract, and multiply matrices of appropriate dimensions. **N-VM.9.** (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **N-VM.10.** (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **N-VM.11.** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **N-VM.12.** (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context.★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.* **A-REI.3.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **A-REI.8. (+)** Represent a system of linear equations as a single matrix equation in a vector variable. **A-REI.9. (+)** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	Inverse Identity Network Transformation	